CHAPTER 4

MEASURES OF POSITION

Skewness

- Skewness measures the lack of symmetry in a data distribution
- The value of skewness falls between – 3.0 and +3.0
- The skewness value of – 3.0 indicates that the distribution is extremely skewed to the left
- The skewness value of + 3.0 indicates that the distribution is extremely skewed to the right
- The skewness value of 0 indicates the distribution is symmetrical
- However, in research, data are considered to be normally distributed if the skewness value falls between – 1.0 to +1.0

a) Mean > Median > Mode : The distribution is positively skewed or is skewed to the right

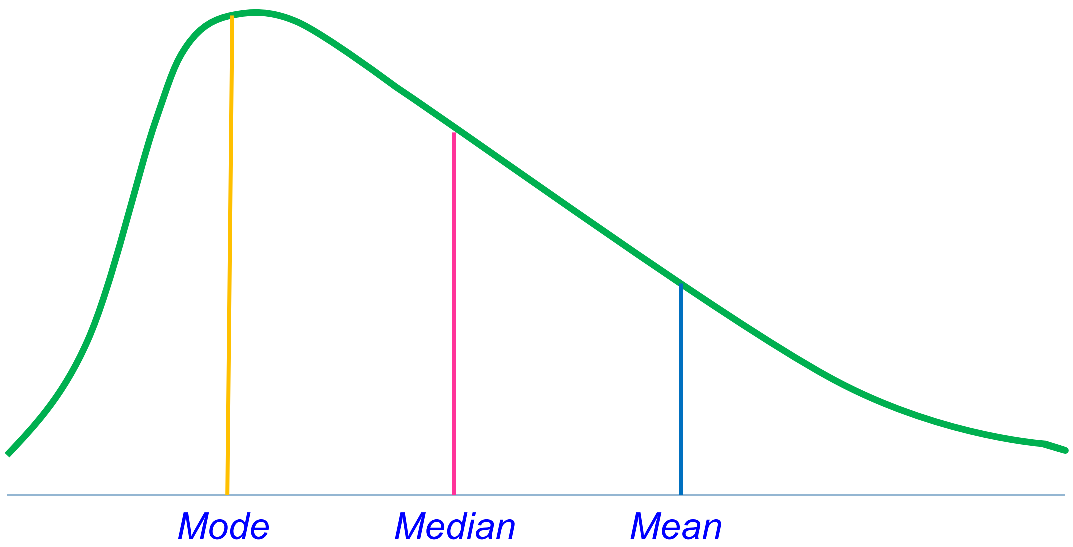
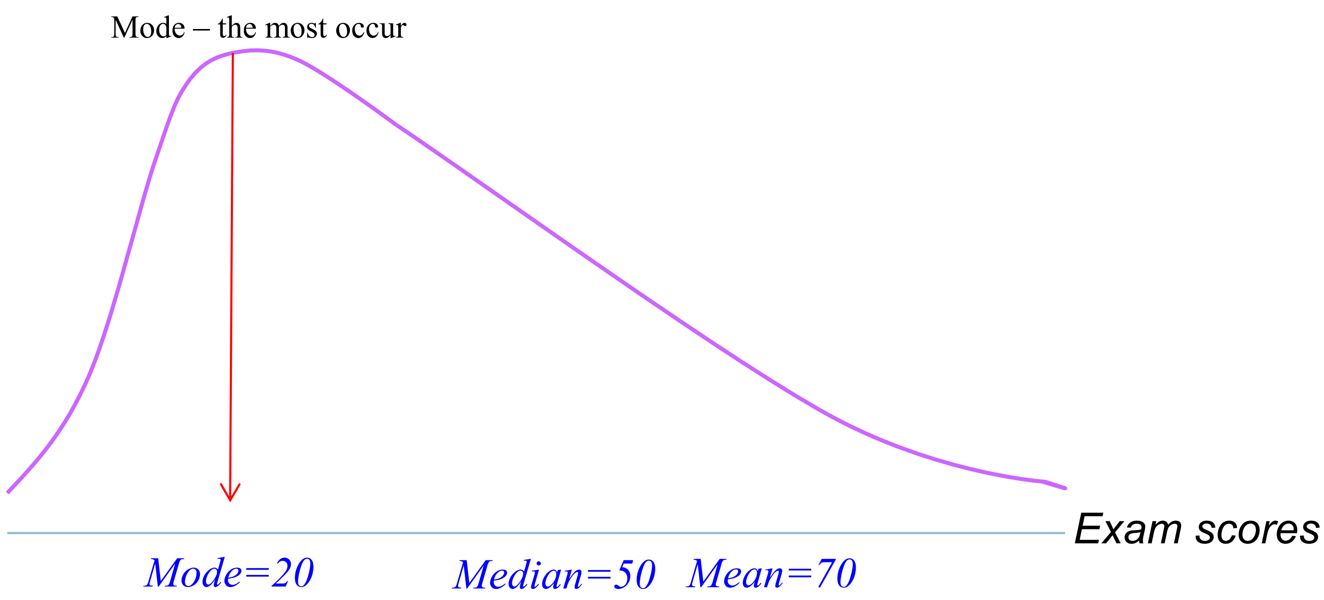


Figure 4.7 Distribution is positively

EXAMPLE – Exam scores

Mean = 70 marks
Median = 50 marks
Mode = 20 marks



The distribution is positively skewed or is skewed to the right

b) Mean = Median = Mode : The distribution is symmetrical

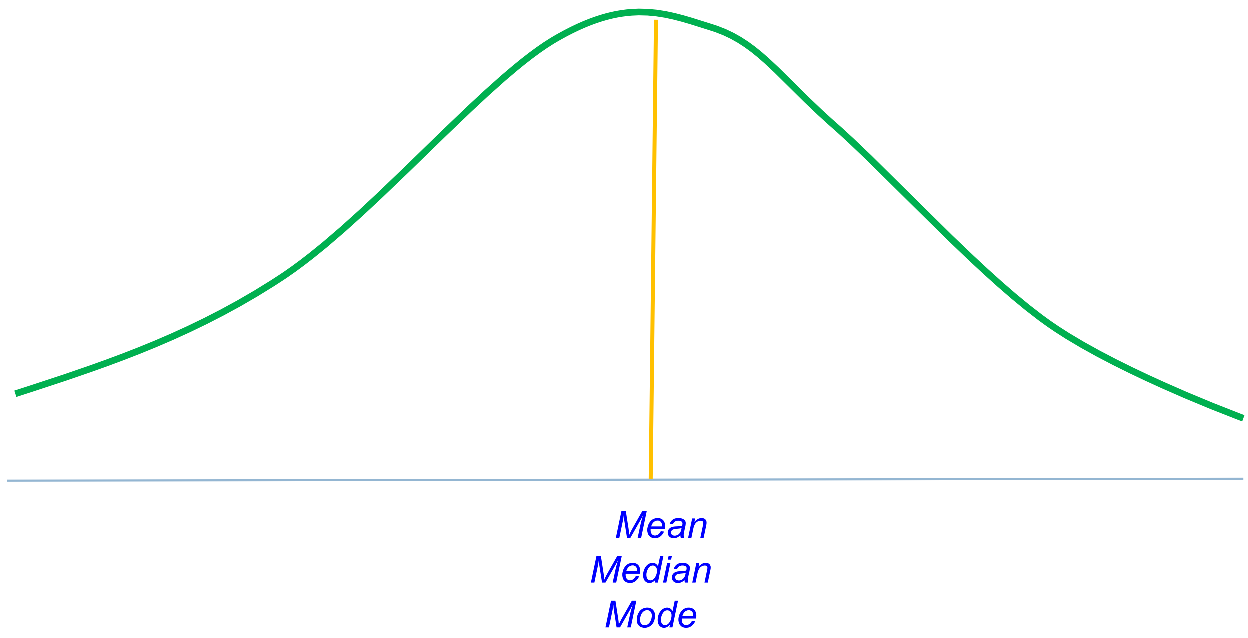


Figure 4.8 Distribution is symmetrical

c) Mean < Median < Mode : The distribution is negatively skewed or is skewed to the left

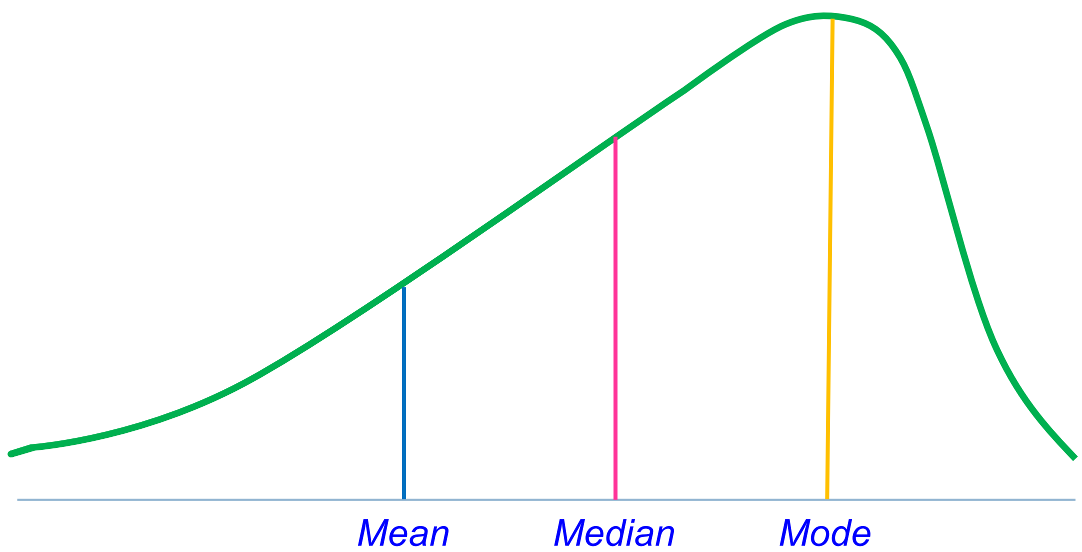
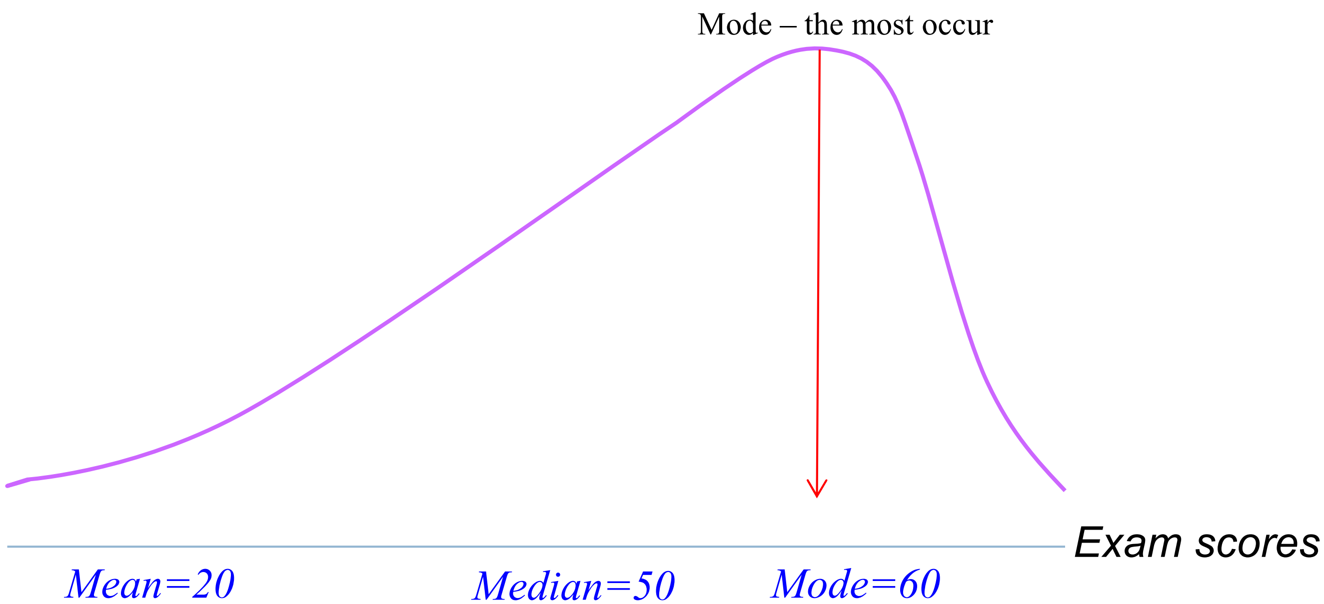


Figure 4.9 Distribution is negatively skewed

EXAMPLE – Exam scores

Mean = 20 marks
Median = 50 marks
Mode = 60 marks

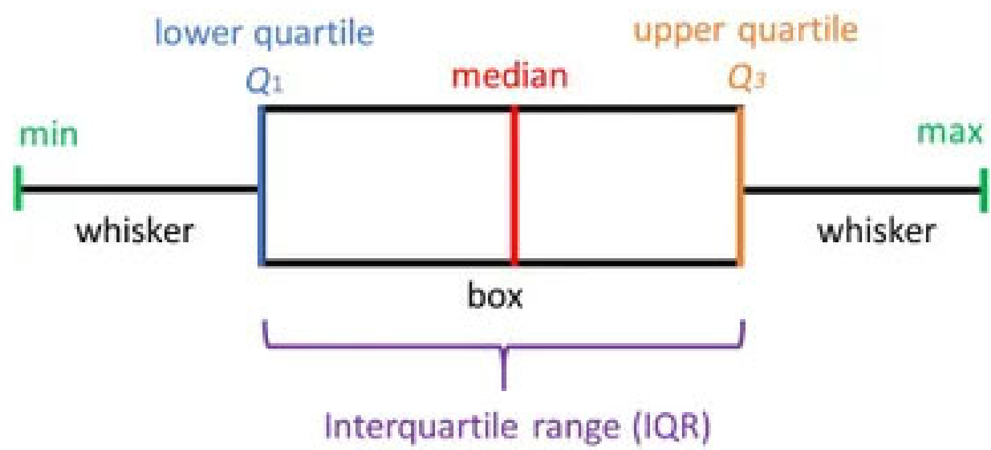


The distribution is negatively skewed or is skewed to the left

MEASURES OF POSITION
(Ungrouped data)

Box and Whisker Plot

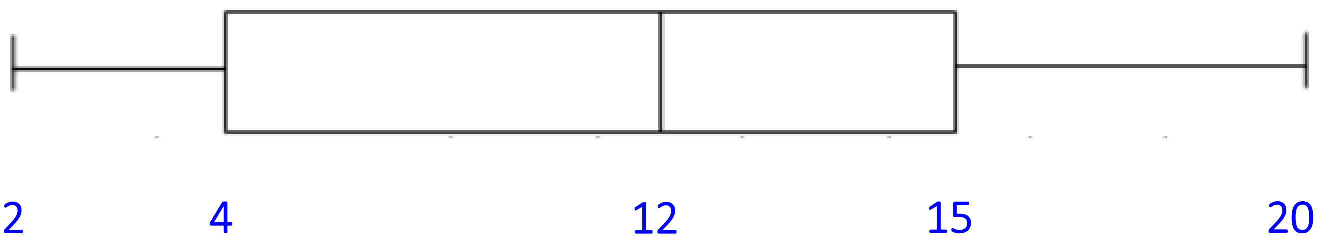
- A box and whisker plot provides useful graphical representation of data using the minimum value, first quartile, second quartile, third quartile and maximum value.



Example 1

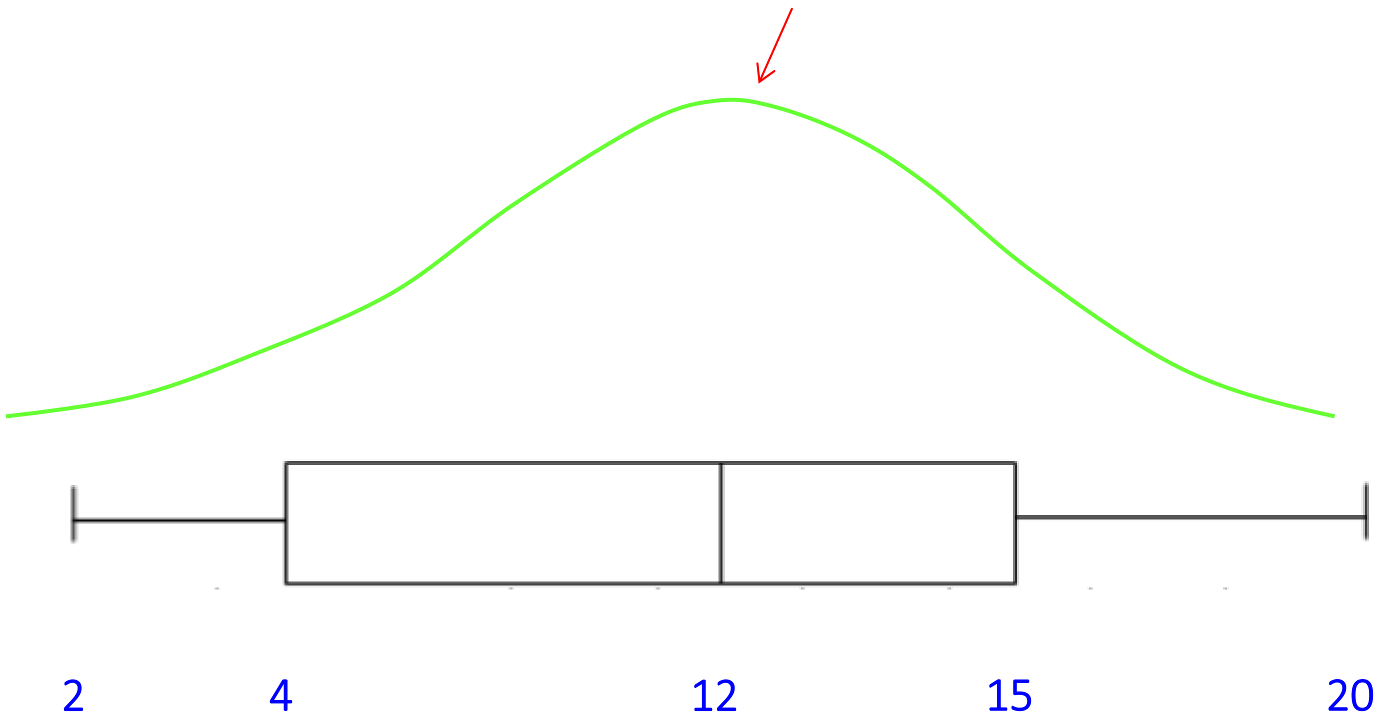
Below represent quiz scores out of 20 points for Quiz 1

Min = 2
2
Q1 = 4
8
Q2 = 12
3
Q3 =15
5
Max = 20



Interpretation

- The distribution is slightly skewed to the right



First quartiles, Q1

- The first quartiles is a proportional value where 25% of the observations are smaller and 75% are larger than the value

Example 4.7 The three year annual returns of 14 low risk funds arranged in ascending order are given as follows.

9.77	11.35	12.46	13.80	15.47	17.48	18.37
18.47	18.61	20.72	21.49	22.47	31.50	38.16

Steps

1) Sort data in ascending order

9.77	11.35	12.46	13.80	15.47	17.48	18.37
18.47	18.61	20.72	21.49	22.47	31.50	38.16

2) Position of first quartile

$$= \frac{n+1}{4}$$

$$= \frac{14+1}{4}$$

$$= 3.75 \sim 4^{th}$$

3) First quartile, Q1 = 13.80

Third quartiles, Q3

- The first quartiles is a proportional value where 75% of the observations are smaller and 25% are larger than the value

Example 4.7 The three year annual returns of 14 low risk funds arranged in ascending order are given as follows.

9.77	11.35	12.46	13.80	15.47	17.48	18.37
18.47	18.61	20.72	21.49	22.47	31.50	38.16

Steps

1) Sort data in ascending order

9.77	11.35	12.46	13.80	15.47	17.48	18.37
18.47	18.61	20.72	21.49	22.47	31.50	38.16

2) Position of third quartile $= \frac{3(n+1)}{4}$

$$= \frac{3(14+1)}{4}$$

$$= 11.25 \sim 11^{th}$$

3) Third quartile, Q3 = 21.49

MEASURES OF POSITION
(Grouped data)

Info

		Formula	Graph
Measures of position	First quartile	$Q_1 = L_1 + \left[\dfrac{\frac{n}{4} - F_1}{f_1} \right] \times C_1$	Ogive
	Third quartile	$Q_3 = L_3 + \left[\dfrac{\frac{3n}{4} - F_3}{f_3} \right] \times C_3$	Ogive

First quartile, Q1

- For grouped data, first quartile is calculated as follows.

$$Q_1 = L_1 + \left[\frac{\frac{n}{4} - F_1}{f_1} \right] \times C_1$$

where

n = sample size

L_1 = lower boundary of the Q1 class

F_1 = cumulative frequency before the Q1 class

f_1 = frequency of the Q1 class

C_1 = Q1 class size

First quartile, Q1

- Using FORMULA

Example 4.12

- The table shows the distribution of test scores obtained by 42 students in a Statistics class

Scores obtained	Number of students
80 – 90	1
90 – 100	2
100 – 110	5
110 – 120	10
120 – 130	15
130 – 140	7
140 – 150	2
Total	42

- Calculate the first quartile,Q1 and explain its meaning.

Step 1: Obtain the cumulative frequencies

Scores obtained	Number of employees (*f*)	Cumulative frequency	
80 – 90	1	1	
90 – 100	2	3	
100 – 110	5	8	
110 – 120	10	18	
120 – 130	15	33	
130 – 140	7	40	
140 – 150	2	42	
Total	42		

Step 2: Obtain the position of data

Scores obtained	Number of employees (f)	Cumulative frequency	Position of data
80 – 90	1	1	1
90 – 100	2	3	2 – 3
100 – 110	5	8	4 – 8
110 – 120	10	18	9 – 18
120 – 130	15	33	19 – 33
130 – 140	7	40	34 – 40
140 – 150	2	42	41 – 42
Total	42		

- Step 3: Obtain the first quartile location

$$\frac{n}{4} = \frac{42}{4} = 10.5^{th} \qquad or \qquad \frac{\sum f}{4} = \frac{42}{4} = 10.5^{th}$$

- Step 4: Obtain the first quartile class

Scores obtained	Number of employees (f)	Cumulative frequency	Position of data
80 – 90	1	1	1
90 – 100	2	3	2 – 3
100 – 110	5	8	4 – 8
110 – 120	10	18	9 – 18
120 – 130	15	33	19 – 33
130 – 140	7	40	34 – 40
140 – 150	2	42	41 – 42
Total	42		

10.5^{th} is between 9 and 18

→ *Q1 class* (row 110 – 120)

- Step 5: Apply the formula

$$Q_1 = L_1 + \left[\frac{\frac{n}{4} - F_1}{f_1} \right] \times C_1$$

$$Q_1 = 110 + \left[\frac{\frac{42}{4} - 8}{10} \right] \times 10$$

$$Q_1 = 112.50$$

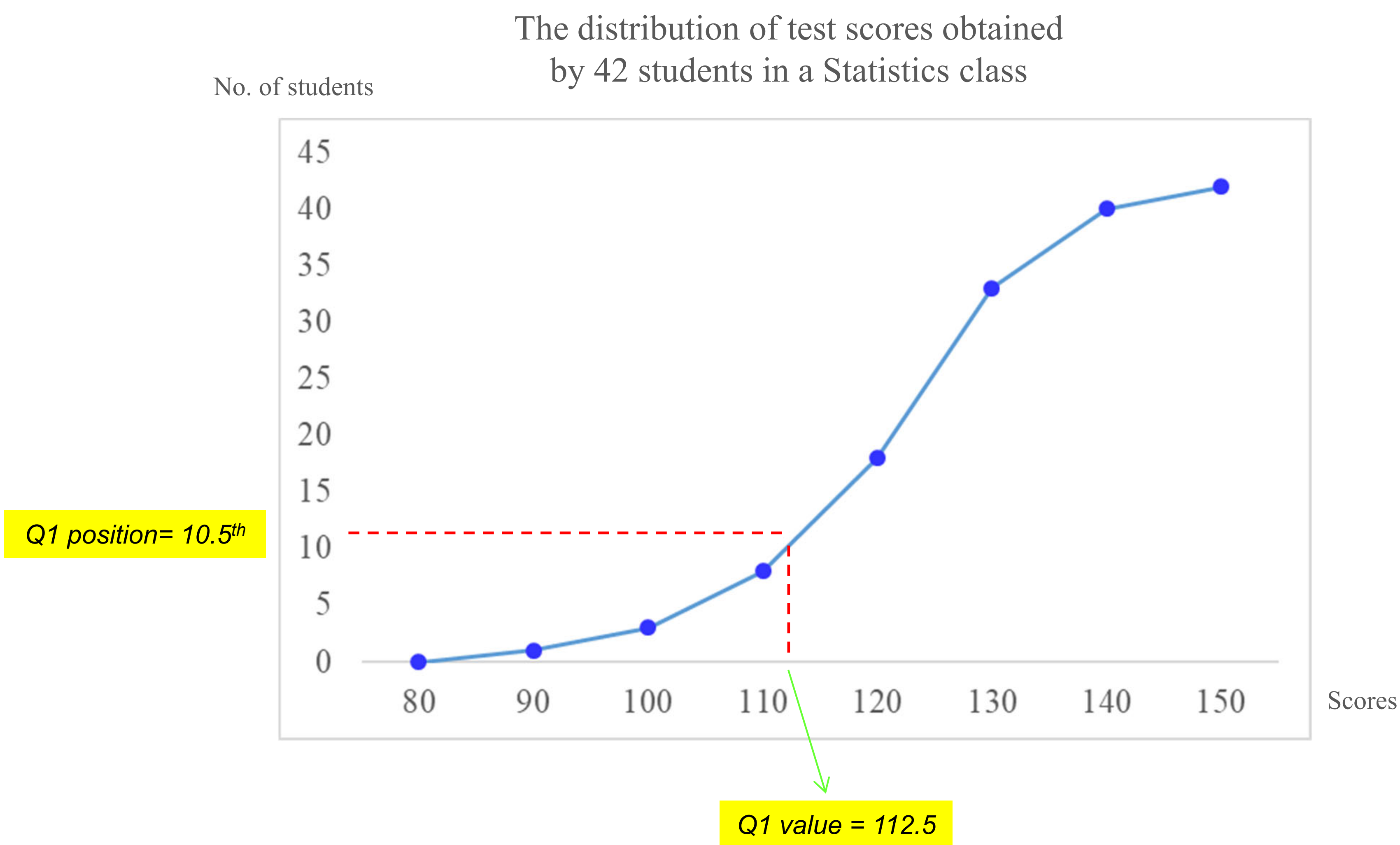
Interpretation

25% of students scored less than 112.5 marks and other 75% scored more than 112.5 marks

First quartile, Q1

- Using OGIVE

Estimating the Q1 from an OGIVE



Third quartile, Q3

- For grouped data, third quartile is calculated as follows.

$$Q_3 = L_3 + \left[\frac{\frac{3n}{4} - F_3}{f_3} \right] \times C_3$$

where

n = sample size

L_3 = lower boundaries of the Q3 class

F_3 = cumulative frequency before the Q3 class

f_3 = frequency of the Q3 class

C_3 = Q3 class size

Third quartile, Q3

- Using FORMULA

Example 4.12

- The table shows the distribution of test scores obtained by 42 students in a Statistics class

Scores obtained	Number of students
80 – 90	1
90 – 100	2
100 – 110	5
110 – 120	10
120 – 130	15
130 – 140	7
140 – 150	2
Total	42

- Calculate the third quartile,Q3 and explain its meaning.

Step 1: Obtain the cumulative frequencies

Scores obtained	Number of employees (*f*)	Cumulative frequency	
80 – 90	1	1	
90 – 100	2	3	
100 – 110	5	8	
110 – 120	10	18	
120 – 130	15	33	
130 – 140	7	40	
140 – 150	2	42	
Total	42		

Step 2: Obtain the position of data

Scores obtained	Number of employees (*f*)	Cumulative frequency	Position of data
80 – 90	1	1	1
90 – 100	2	3	2 – 3
100 – 110	5	8	4 – 8
110 – 120	10	18	9 – 18
120 – 130	15	33	19 – 33
130 – 140	7	40	34 – 40
140 – 150	2	42	41 – 42
Total	42		

- Step 3: Obtain the third quartile location

$$\frac{3(n)}{4} = \frac{3(42)}{4} = 31.5^{th} \quad or \quad \frac{3\sum f}{4} = \frac{3(42)}{4} = 31.5^{th}$$

- Step 4: Obtain the third quartile class

31.5^{th} is between 19 and 33

Scores obtained	Number of employees (*f*)	Cumulative frequency	Position of data
80 – 90	1	1	1
90 – 100	2	3	2 – 3
100 – 110	5	8	4 – 8
110 – 120	10	18	9 – 18
120 – 130	15	33	19 – 33 → *Q3 class*
130 – 140	7	40	34 – 40
140 – 150	2	42	41 – 42
Total	42		

- Step 5: Apply the formula

$$Q_3 = L_3 + \left[\frac{\frac{3(n)}{4} - F_3}{f_3} \right] \times C_3$$

$$Q_3 = 120 + \left[\frac{\frac{3(42)}{4} - 18}{15} \right] \times 10$$

$$Q_3 = 129$$

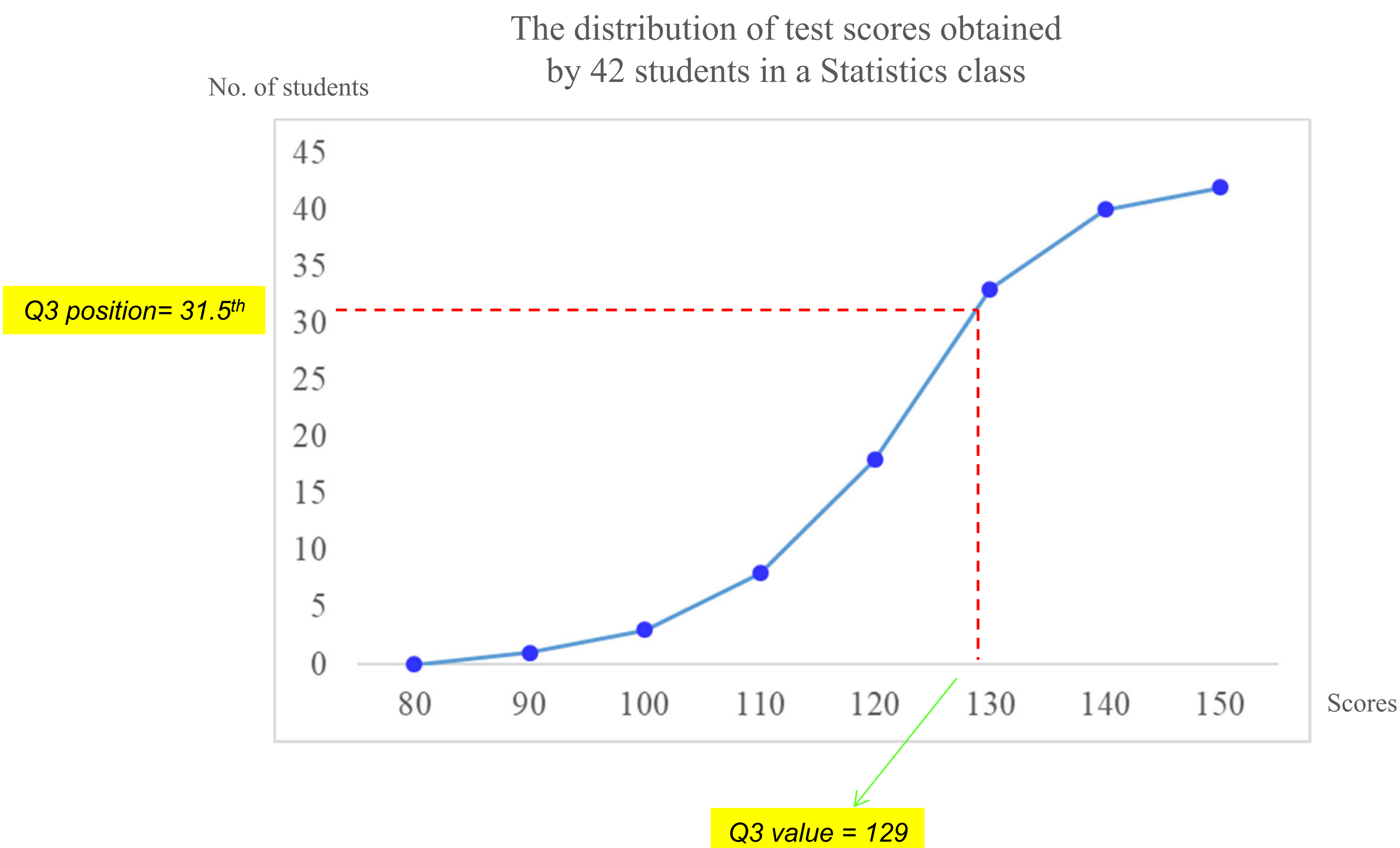
Interpretation

75% of students scored less than 129 marks and other 25% scored more than 129 marks

Third quartile, Q3

- Using OGIVE

Estimating the Q3 from an OGIVE



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