UNIVERSITI TEKNOLOGI MARA

UiTM di hatiku

COMPOUND INTEREST

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LEARNING OUTCOMES

By the end of this chapter, student should be able to :

- explain the concepts of time value of money,
- use the compound amount formula to find the future value, compound interest, and present value of investments and loans.

WHAT IS COMPOUND INTEREST?

Definition: *Interest that is earned on both the principal and any interest that has been earned previously*

EXAMPLE

RM100 is deposited into a savings account that earns 5% interest compounded annually. The table shows the balance of the account after each of 5 years.

Years	Principal Amount	Rate of Interest	Interest	Compound Amount
1	\$100.00	5%	\$100.00 × 0.05 = \$5.00	\$100.00 + \$5.00 = \$105.00
2	\$105.00	5%	\$105.00 × 0.05 = \$5.25	\$105.00 + \$5.25 = \$110.25
3	\$110.25	5%	\$110.25 × 0.05 = \$5.51	\$110.25 + \$5.51 = \$115.76
4	\$115.76	5%	\$115.76 × 0.05 = \$5.79	\$115.76 + \$5.79 = \$121.55
5	\$121.55	5%	\$121.55 × 0.05 = \$6.08	\$121.55 + \$6.08 = \$127.63

COMPOUND INTEREST FORMULA

The mathematical formula for calculating compound interest depends on several factors. These factors include the amount of money deposited called the principal, the annual interest rate (in decimal form), the number of times the money is compounded per year, and the number of years the money is left in the bank. These factors lead to the formula

$$S = P(1+i)^n \quad where \quad S = \text{Compound Amount}$$

$$P = \text{Principal}$$

$$i = \frac{\text{k}}{\text{m}}$$

$$n = \text{mt}$$

Thus, the compound interest can be calculated by subtracting the compound amount, S from the initial principal, P

$$I = S - P$$

$$S = \text{Compound Amount}$$

$$P = \text{Principal}$$

Note: *if interest is 3.5% compounded monthly, the k = 3.5% and m = 12*

DIFFERENCE BETWEEN P AND S

P (left)	S (right)
Amount in the beginning	Amount at the end
Initial savings/investment	Accumulated amount
Loan	Total repayment

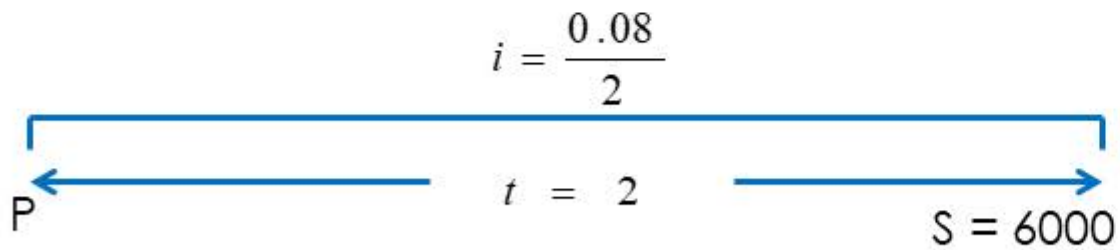
DETERMINING M

k% compounded m	m
annually (every year)	1
semi-annually (every six months)	2
every four months	3
quarterly (every three months)	4
every two months	6
monthly (every month)	12
daily (everyday)*	360

* Banker's Rule

EXAMPLE

Stephen wishes to have RM6,000 in two years time to tour Middle East. How much should he invest today at 8% compounded semi-annually?



SOLUTION

$$S = 6000$$

$$i = \frac{0.08}{2}$$

$$n = 2(2) = 4$$

$$S = P(1+i)^n$$

$$6000 = P\left(1+\frac{0.08}{2}\right)^4$$

$$P = RM\ 5128\ .83$$

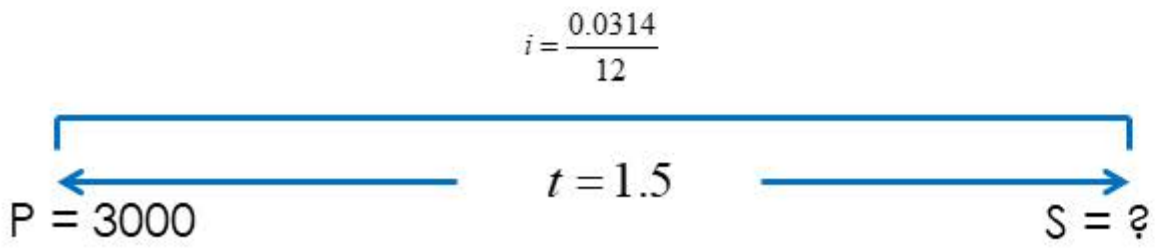
Round your final answer to 2 decimal places

Example

Rahimah took a loan of RM3,000 on 9 November 2012 at 3.14% interest compounded monthly for one and a half years.

i) Determine the date of repayment.

ii) Find the maturity value of the loan.



SOLUTION

i)

$$9\ \text{Nov}\ 2012 + 1\ \text{year} = 9\ \text{Nov}\ 2013$$

$$9\ \text{Nov}\ 2013 + 6\ \text{months} = 9\ \text{May}\ 2014$$

ii) $P = 3000$

$$i = \frac{0.0314}{12}$$

$$n = 12\,(1.5) = 18$$

$$S = P(1+i)^n$$

$$= 3000\left(1 + \frac{0.0314}{12}\right)^{18}$$

$$= RM\ 3144\ .49$$

EXAMPLE

RM5,000 was invested for four years. The bank offered 6% compounded monthly for the first three years and r% compounded annually for the rest of the period. If the amount in the account at the end of the term was RM7,000, find r.

SOLUTION

$$S = P(1+i)^n$$

$$= 5000\left(1+\frac{0.06}{12}\right)^{36}$$

$$= \text{RM}5983.40$$

$$S = P(1+i)^n$$

$$7000 = 5983.40\left(1+\frac{r}{1}\right)^1$$

$$r = 16.99\%$$

P = 5000

$i = \frac{0.06}{12}$

$i = \frac{r}{1}$

S = 7000

EXAMPLE

Bobby invested RM8000 in an investment account at an interest rate of 6% compounded quarterly. Three years later he withdrew half of the initial investment from the account. Calculate the amount left in Bobby's account just after the withdrawal.

SOLUTION

$$P = 8000$$

$$i = \frac{0.06}{4}$$

$$n = 4(3) = 12$$

$$S = P(1+i)^n$$

$$= 8000\left(1 + \frac{0.06}{4}\right)^{12}$$

$$= \text{RM}9564.95$$

$$\text{Half of the initial investment} = \frac{1}{2}(8000) = 4000$$

$$\text{Amount left} = 9564.95 - 4000$$

$$= \text{RM}5564.95$$

EXAMPLE

EXAMPLE

Arnold opened a savings account which offered an interest rate of 10% compounded quarterly with an initial deposit of RM2000. Two years later, he deposited RM3000 into the same account. Find the amount accumulated four years after the initial deposit.

SOLUTION

$$P_1 = 2000$$
$$n_1 = 4(2) = 8$$
$$n_2 = 4(2) = 8$$
$$i = \frac{0.1}{4}$$

$$S = P_1(1+i)^{n_1}$$
$$= 2000\left(1+\frac{0.1}{4}\right)^{8}$$
$$= \text{RM}2436.81$$

$$P_2 = 2436.81 + 3000$$
$$= 5436.81$$

$$S = P_2(1+i)^{n_2}$$
$$= 5436.81\left(1+\frac{0.1}{4}\right)^{8}$$
$$= \text{RM}6624.23$$

NOTES

If there any changes happened, stop, and find the value needed before proceed to other steps.

- P ---> saved, invested, borrowed, deposited
- S ---> total payment, total savings, accumulated amount

END OF LEARNING SLIDES
HOPE YOU HAVE MASTER THE MATERIALS COVERED
GOOD LUCK & ALL THE BEST