ANNUITY

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LEARNING OUTCOMES

By the end of this chapter, student should be able to :

- find the future value of annuity,
- find the present value of annuity,
- solve for annuity payment, R, the number of payments, n, and the interest rate, I, and
- identify the problems where the present value and the future value of the annuity formula can be appropriately applied.

WHAT IS ANNUITY?

- An **annuity** is a sequence of payments made at regular time intervals. The time period in which these payments are made is called the **term** of the annuity while **payment period** is interval between annuity payments.
- An annuity in which the payments are made at the *end* of each payment period is called an **ordinary annuity**
- An annuity in which the payment period coincides wit the interest compounding period is called a **simple annuity**
- Annuities considered here have terms given by fixed time intervals, periodic payments equal in size, payments made at the end of the payment periods, and payment periods coincide with the interest compounding period.
- Example of annuity includes house rents, mortgage payments, instalment payments on automobiles, and interest payments on money invested.

FUTURE VALUE OF AN ANNUITY

The **future value *S* of an annuity** of *n* payments of *R* ringgits each, paid at the end of each investment period into an account that earns interest at the rate of *i* per period, is given by

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

Then, to find the amount of interest paid or earned can be found by subtracting the total amount paid from the future value, i.e.,

$$I = S - nR$$

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EXAMPLE

Suneeta intends to invest RM1,500 every month for 3 years at 12% compounded monthly. Find

i) the future value at the end of the annuity

ii) the interest that will be earned

SOLUTION

$$S = R\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 1500\left[\frac{\left(1+\frac{0.12}{12}\right)^{36} - 1}{\frac{0.12}{12}}\right]$$

$$= RM64615.32$$

$$I = S - Rn$$

$$= 64615.32 - 1500(36)$$

$$= RM10615.32$$

PRESENT VALUE OF AN ANNUITY

- The **present value *P* of an annuity** of *n* payments of *R* dollars each, paid at the end of each investment period into an account that earns interest at the rate of *i* per period, is

$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

Then, to find the amount of interest paid or earned can be found by subtracting the present value from the total amount paid, i.e.,

$$I = nR - A$$

EXAMPLE

Find the present value of an annuity with periodic payments of RM2000 every 6 months, for a period of 10 years at an interest rate of 6% compounded semi-annually. Find the total interest charged.

SOLUTION

$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 2000\left[\frac{1-\left(1+\frac{0.06}{2}\right)^{-20}}{\frac{0.06}{2}}\right]$$

$$= RM29754.95$$

$$I = Rn - A$$

$$= 2000(20) - 29754.95$$

$$= RM10245.05$$

APPLICATION OF ANNUITY

The application of annuity include the concept of taking a loan from a financial institution, purchasing item through instalment, etc.

Consider a scenario where you want to buy a house worth RM250,000 cash price from a house developer. Since you don't have that much money, a bank offers a loan, 90% of the cash price at certain interest rate paid through equal payment for certain duration of time. Thus, in this case you will have to pay the developer 10% of the cash price as down payment, while the balance 90% will be paid by the bank to the developer for the remaining amount. All you need to do now is to repay the loan to the bank by instalment.

Down Payment (DP) = % x Cash Price (CP) = 10% x 250,000 = 25,000
Balance (A) = CP – DP = 250,000 – 25,000 = **RM 225,000** $\leftarrow$ This is the amount that you have to loan from a bank

Two important concepts to remember. ❶ *Late Payment* $\rightarrow$ *use formula S*
Hint : missed payment or failed to pay arrears

❷ *Early Payment* $\rightarrow$ *use formula A*
Hint : settle the debt immediately

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EXAMPLE

The cash price of a house is RM250,000. Steven paid a down payment of 10% and took a loan at 3.5% compounded monthly to finance the balance. The loan is to be repaid over 25 years by monthly installments.

i) Find the amount of monthly payment.

ii) If Steven failed to make the first 5 monthly payments, how much should he pay on the 6th payment including the arrears?

SOLUTION

$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$225{,}000 = R\left[\frac{1-\left(1+\frac{0.035}{12}\right)^{-300}}{\frac{0.035}{12}}\right]$$

$$R = RM1{,}126.40$$

$$S = R\left[\frac{(1+i)^{n}-1}{i}\right]$$

$$= 1{,}126.40\left[\frac{\left(1+\frac{0.035}{12}\right)^{6}-1}{\frac{0.035}{12}}\right]$$

$$= RM\,6{,}807.87$$

EXAMPLE

Ranjit makes a loan from a bank for 20 years. An interest rate of 7.5% compounded monthly with a monthly payment of RM100 is charged by the bank towards the loan. An amount of RM5,000 that is part of the loan is invested in Scheme A that offers simple interest rate of 5% per annum.

i) Find the future value of the investment in Scheme A after 10 years.

ii) Ranjit intends to settle the debt immediately at the end of the 10th year by using the money obtained from his investment in i). How much more money should he add?

SOLUTION

$$i)\ P=5,000;\ r=0.05;\ t=10$$

$$S=P(1+rt)$$

$$=5000\,(1+0.05(10))$$

$$=RM\ 7,500$$

ii) To settle the loan after 10 years

$$R=100;\ i=\frac{0.075}{12};\ n=12(10)=120$$

Find A,

$$A=R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$=100\left[\frac{1-\left(1+\frac{0.075}{12}\right)^{-120}}{\frac{0.075}{12}}\right]$$

$$=RM\ 8,424.47$$

So, additional amount is

$$=8424.47-7500=RM\ 924.47$$

EXAMPLE

Amsyari borrowed RM80,000 from a bank for 9 years at the interest of 11.5% compounded monthly.

i) Find the monthly repayment.

ii) Calculate the total interest paid for the loan.

iii) Determine the 3rd repayment amount to settle all the arrears if he has not paid the first 2 repayments.

iv) Immediately after the 90th payment, Amsyari decided to settle the loan. Calculate the amount of the settlement.

SOLUTION

$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$80000 = R\left[\frac{1-\left(1+\frac{0.115}{12}\right)^{-108}}{\frac{0.115}{12}}\right]$$

$$R = RM1{,}192.29$$

$$I = Rn - A$$
$$= 1192.29(108) - 80000$$
$$= RM48767.32$$

$$S = R\left[\frac{(1+i)^{n}-1}{i}\right]$$

$$= 1192.29\left[\frac{\left(1+\frac{0.115}{12}\right)^{3}-1}{\frac{0.115}{12}}\right]$$

$$= RM\,3{,}611.26$$

$$n = 108 - 90 = 18$$

$$A = R\left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$= 1192.29\left[\frac{1-\left(1+\frac{0.115}{12}\right)^{-18}}{\frac{0.115}{12}}\right]$$

$$= RM\,19{,}626.16$$

EXAMPLE (COMPOUND INTEREST + ANNUITY)

RM100 was invested at the end of every 3 months for 5 years. After 5 years, no more deposit was made. The interest rate is 6% compounded quarterly.

i) Find the amount in the account at the end of 7 years.

ii) If 77% of the amount in the account has been withdrawn at the end of 7 years, find the balance left in the account just after the withdrawal.

SOLUTION

i) For the first 5 years (Annuity)

$$R=100; m=4; i=\frac{0.06}{4}; n=20$$

$$S=R\left[\frac{(1+i)^n-1}{i}\right]$$

$$=100\left[\frac{\left(1+\frac{0.06}{4}\right)^{20}-1}{\frac{0.06}{4}}\right]$$

$$=RM\,2{,}312.37$$

For next 2 years (compounded)

$$S=P(1+i)^n$$

$$=2312.37\left(1+\frac{0.06}{4}\right)^{8}$$

$$=RM\,2{,}604.86$$

ii) Amount of withdrawal

$$=\frac{77}{100}(2604.86)$$

$$=RM\,2{,}005.74$$

Amount left

$$=2604.86-2005.74$$

$$=\text{RM}\,599.12$$

EXERCISE (COMPOUND INTEREST + ANNUITY)

When Victor was 30 years old, he started to deposit RM150 at the end of every month in an account that pays 2% compounded monthly. However, 5 years later he decided to withdraw RM1,000. Then, he left the account untouched until he reached 50 years old. Calculate the total amount of money in the account when he was 50 and the total interest earned for this investment.

Try this question and check your answer with your instructor.

END OF LEARNING SLIDES
HOPE YOU HAVE MASTER THE MATERIALS COVERED
GOOD LUCK & ALL THE BEST